Beam Debunching in the SNS Superconducting Cavity Linac

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Introduction

The purpose of this note is to estimate the debunching of a coasting RF-modulated beam in the SNS superconducting cavity Linac (SCL) durng commissioning, when the beam drifts for very long distances without any longitudinal restoring force to counteract the space charge forces.

Debunching refers to the elongation (phase spread) of the bunches, leading to loss of the RF amplitude modulation of the beam current, especially the higher frequency harmonics. These components, especially the 402.5 MHz and the 805 MHz components, are important for the proper operation of the RF beam position monitors and beam excitation of the 805 MHz SCL cavities during commissioning.

A Gaussian beam bunch of rms width s can be represented in the time domain as a beam current I(t):

$$I(t) = \frac{I_0}{\sqrt{2p}\, s} \cdot \exp\left[\frac{-t^2}{2s^2}\right] \tag{1}$$

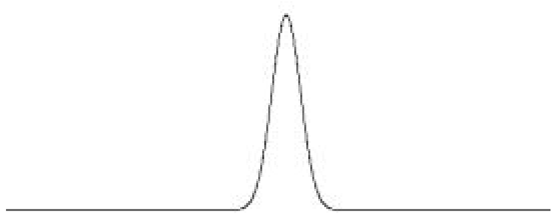

Figure 1. Single Gaussian beam bunch

The beam bunch can also be represented as a Fourier series summation over all the harmonic frequencies. If the beam bunches are periodic with a period $2\pi/w$, then the normalized amplitude of the n^{th} Fourier harmonic of the fundamental frequency $w/2p$ is given by [1]

$$A_n = \exp\left[-n^2 w^2 s^2 / 2\right] \tag{2}$$

For a beam with average current I_0, the time dependent beam current $I(t)$ may be expressed as a summation over all Fourier harmonics of the fundamental frequency, using the normalized amplitudes calculated in Eq (2).

$$I(t) = I_0 + 2I_0 \cdot \sum_{n=1}^{\infty} A_n \cdot \cos(nwt) \tag{3}$$

Thus if s = 100 ps, and $w/2\pi$ = 402.5 MHz, then the normalized amplitudes of the first 5 Fourier harmonics are

Table 1. Normalized Fourier harmonic amplitudes for a 100-ps rms-width beam bunch

n	Frequency	Amplitude A_n
1	402.5 MHz	0.969
2	805	0.880
3	1207.5	0.750
4	1610	0.599
5	2012.5	0.450

For very short beam bunches, all harmonics have $A_n = 1$.

Figure 2 shows the Fourier series expansion for s =100-ps width, f =402.5-MHz periodic bunches.

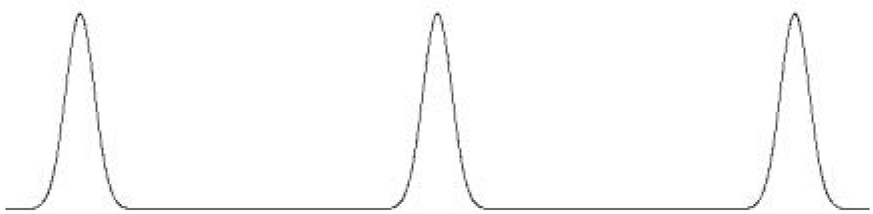

Figure 2. 402.5 MHz periodic bunches with 100 ps rms width.

Thus, either Eq(1) or Eq(3) may be used to provide a time-domain representation of the beam bunch shape. Eq(3) in addition provides information on the amplitudes of the individual harmonics of the bunching frequency.

Momentum Spread and Debunching

Proton beam bunches in an accelerating structure are confined longitudinally with RF focusing fields that counteract the space charge forces. When the accelerating field is turned off and the beam drifts, the space charge forces cause the beam bunch to expand longitudinally. The longitudinal expansion is due to a space-charge-induced energy spread, which is of the order of 450 keV (approximate empirical value), full width at base. This is a very approximate number and varies perhaps by +/- 50% or more from case to case. This energy spread develops very quickly after the rf restoring force is removed, resulting in an asymptotic value for the energy spread that changes very little as the beam drifts.

Because of the energy spread, there is a velocity spread, which is the source of the phase spread. Thus the phase spread increases nearly linearly with the drift distance. The rate of the increase in phase spread is very dependent on the beam energy.

The energy spread is very nearly parabolic, rather than Gaussian, so the beam bunch maintains a parabolic shape as the beam drifts.

For a beam of kinetic energy *E* and an energy spread *DE*, The phase spread *W*(*x*) for a drift distance x is (see Appendix)

$$W(x) = \frac{x}{c} \cdot \frac{1}{b^3 g^3} \cdot \frac{\Delta E}{Mc^2} \tag{4}$$

This is plotted in Figure 3 for E=185 MeV and ΔE = 0.45 MeV.

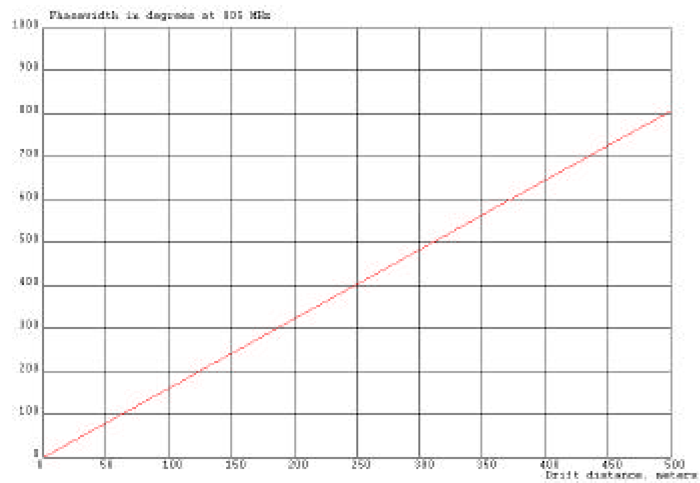

Figure 3. Phasewidth (degrees) at 805 MHz vs. drift distance x for *E*= 185 MeV.

For a parabolic bunch shape with full width *W* at the base, the Fourier harmonic amplitude of the nth harmonic is [1]:

$$A_n = 3\left[\frac{\sin(a)}{a^3} - \frac{\cos(a)}{a^2}\right] \qquad \text{where } a = npWf \tag{5}$$

Figure 4 shows the Fourier series expansion using Eq(3) and Eq(5).

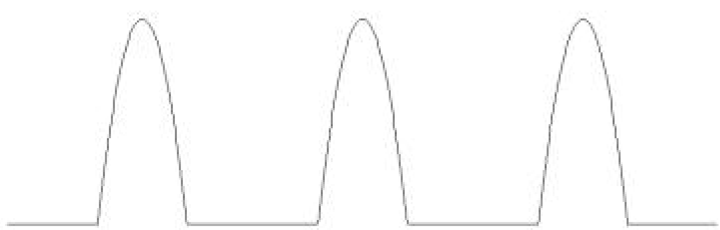

Figure 4. 402.5-MHz Parabolic beam bunches with 1-ns full width at base.

By substituting *W*(*x*) given by Eq(4) into Eq(5), we get the amplitudes of the Fourier harmonics as a function of drift distance x. In Figure 5, the amplitude of the 402.5 MHz and 805 MHz components are plotted as a function of x, for E=185 MeV and ΔE = 0.45 MeV.

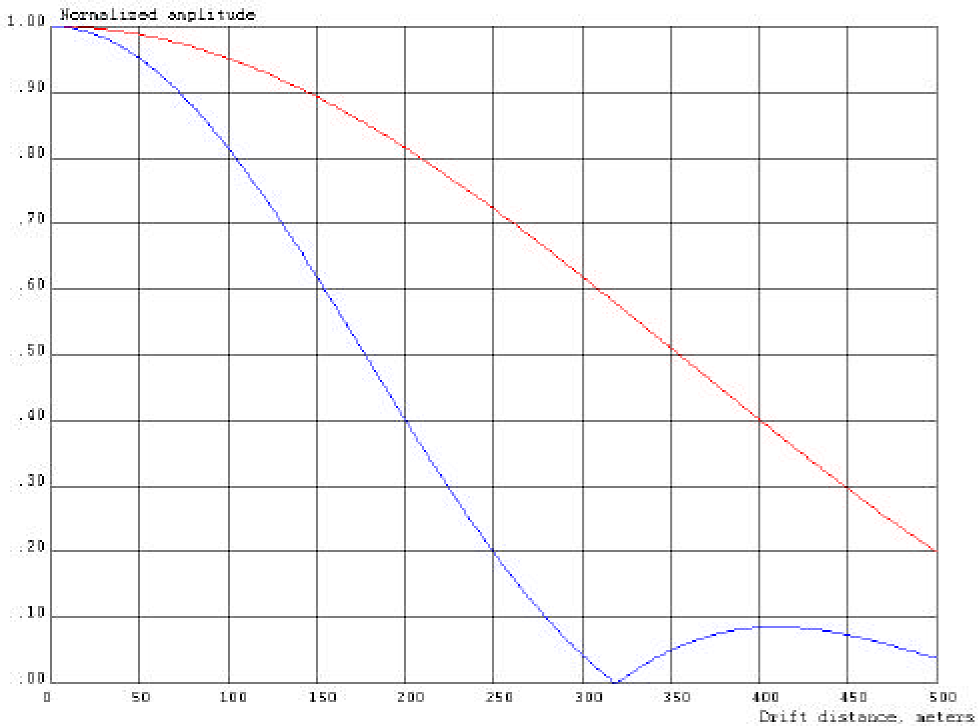

Figure 5. Amplitude of the 402.5-MHz harmonic (top, red) and the 805-MHz harmonic (bottom, blue) as a function of drift distance.

In SNS, the superconducting linac (SCL) is about 230 meters long (including the energy upgrade), and the drift to the linac beam stop is another 80 meters, so the 185 MeV beam, at the entrance to the SCL, would have to drift about 310 meters to the beam stop if all the SCL cavities were unpowered. Figure 5 implies that the 402.5 MHz component, important for operation of the beam position monitors, will not debunch.

When the phase width *W(x)* is about 1.4 RF periods (phase width about 510 degrees), the Fourier harmonic amplitude for that frequency is about zero. Thus, the debunching length *L* for a frequency harmonic *f* is approximately

$$L = 1.4\,\frac{c}{f}\cdot b^3 g^3\cdot\frac{Mc^2}{\Delta E} \qquad (5)$$

Using c=3x10^8 m/s, *M*c^2=939.28 MeV, *f*=402.5 and 805 MHz, and *DE*=450 keV, the debunching lengths as a function of energy *E* are about

E(MeV)	402.5 MHz	805 MHz
20	20 meters	10 meters
50	80	40
100	230	115
200	700	350

500	3400	1700
1000	12,800	6,400

These numbers are very approximate, but do show the rapid scaling of the debunching length with beam energy. PARMELA has been used in the time domain for calculating the debunching length in LEDA [2], and should be used for quantitative results in the SNS.

During commissioning, it is planned to coast the bunched beam through unpowered SC cavities to calibrate the RF pickup phase and amplitude response. From Figure 5, the loss of the 805-MHz amplitude in 8 meters (the length of a period in the SC linac) is negligible.

<u>Conclusion</u>

The above results imply that debunching and loss of the 402.5-MHz Fourier component of a 185-MeV bunched beam will not be a problem during commissioning of the superconducting linac.

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[1] R. E. Shafer, 1989 Beam Instrumentation Workshop Proceedings (BNL), Table 4.1.

[2] Lloyd Young, private communication.

Appendix – Energy Spread and Transit Time Spread

The relation between kinetic energy E and velocity bc in a relativistic particle beam is given by

$$E = (g - 1) \cdot Mc^2 = \frac{Mc^2}{\sqrt{1 - b^2}} - Mc^2 \qquad \text{(A-1)}$$

Thus

$$\frac{dE}{db} = bg^3 \cdot Mc^2 \qquad \text{(A-2)}$$

The transit time for a beam of velocity bc to travel a distance L is

$$t = \frac{L}{bc} \qquad \text{(A-3)}$$

Thus

$$\frac{db}{dt} = \frac{-b^2 c}{L} \qquad \text{(A-4)}$$

Thus the relation between energy spread dE and transit time spread dt is

$$\frac{dE}{dt} = \frac{dE}{db} \cdot \frac{db}{dt} = -bg^3 \cdot Mc^2 \cdot \frac{b^2 c}{L} = -b^3 g^3 Mc^2 \frac{c}{L} \qquad \text{(A-5)}$$

Thus the full width $W(x)$ of a beam bunch with a constant, full width energy spread ΔE drifting a distance x is

$$W(x) = \frac{1}{b^3 g^3} \frac{\Delta E}{Mc^2} \frac{x}{c} \quad \text{seconds} \qquad \text{(A-6)}$$

This does not include the initial width due to longitudinal emittance.